# AVERAGING THE RADIATION TRANSPORT EQUATIONS IN SOLVING TWO-DIMENSIONAL RADIATION-GASDYNAMIC PROBLEMS

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Different methods of averaging the radiation transport equations are considered in solving two-dimensional nonstationary radiation-gasdynamic problems.

The transport equation in the case of local thermodynamic equilibrium and in the absence of scattering has the following form:

$$\frac{\partial I_\varepsilon}{\partial s} = -k_\varepsilon\left(I_\varepsilon - \frac{B_\varepsilon}{\pi}\right), \text{ where } B_\varepsilon = \frac{15}{\pi^4}\frac{\sigma\varepsilon^3}{\exp(\varepsilon/T)-1}. \quad (1)$$

The quantity $k_\varepsilon$ is often dependent in a complex manner on its arguments $\varepsilon$, T and on the gas density $\rho$. The radiation spectrum reflects the characteristic features of the change in $k_\varepsilon$ at different points of space.

The angular distribution of radiation can also be complex in nature. Thus, in the presence of hot "nuclei" in the radiating domain, the radiation intensity $I_\varepsilon$ at this point grows strongly for rays incident at such nuclei as compared with the rays passing through them. As the characteristic dimension of the hot domain changes, the angular radiation distribution also changes.

In nonstationary radiation-gasdynamic problems, Eq. (1) must be solved multiply for different rays passing through each point of space, and for radiation of different wavelengths. An enormous amount of computational work is needed to determine the radiation field when the spectral transport equations are integrated in each time layer. However, the gasdynamic processes are governed only by quantities integrated over the spectrum and the angular variable is governed by the radiation flux. The very nature of the spectral and angular radiation distributions often changes comparatively slowly with time. This permits utilization of methods of averaging the radiation transport equations — integrating them with respect to the angles and (or) the frequencies by using the true spectral and angular radiation distribution found at the time of the averaging.

1. Averaging with respect to the angular variable in one-dimensional problems is performed in a quasidiffusion method [1], a flux variation of quasidiffusion [2], and in the method of mean fluxes [3, 4]. The quasidiffusion method is even extended to the two-dimensional case in [1]. It is expedient to use the two-dimensional analog [5] of the method of mean flux [3, 4] for substantially nonequilibrium radiation with strong anisotropy. For the axial symmetry case, Eq. (1) takes the form

$$\frac{\partial I_\varepsilon}{\partial z}\cos\theta + \frac{\partial I_\varepsilon}{\partial r}\sin\theta\cos\varphi - \frac{1}{r}\frac{\partial I_\varepsilon}{\partial\varphi}\sin\theta\sin\varphi = -k_\varepsilon\left(I_\varepsilon - \frac{B_\varepsilon}{\pi}\right). \quad (2)$$

The unilateral spectral radiation flux densities in the radial $q^{\pm}_{r\varepsilon}$ and axial $q^{\pm}_{z\varepsilon}$ directions are determined by the following expressions:

$$q^{\pm}_{r\varepsilon} = \int\limits_{\omega^{\pm}_r} I_\varepsilon \sin\theta\cos\varphi d\Omega, \quad q^{\pm}_{z\varepsilon} = \int\limits_{\omega^{\pm}_z} I_\varepsilon\cos\theta d\Omega, \quad d\Omega = \sin\theta d\theta d\varphi, \quad (3)$$

where the domains of integration are

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$$\omega_r^+ : -\pi/2 < \varphi < \pi/2, \quad 0 < \theta < \pi, \quad \omega_r^- : \pi/2 < \varphi < 3\pi/2, \quad 0 < \theta < \pi,$$

$$\omega_z^+ : 0 < \varphi < 2\pi, \quad 0 < \theta < \pi/2, \quad \omega_z^- : 0 < \varphi < 2\pi, \quad \pi/2 < \theta < \pi.$$

Let us introduce the functions $\psi_{r\varepsilon}^{\pm}$ and $\psi_{z\varepsilon}^{\pm}$, characterizing the directivity of the radiation:

$$\psi_{r\varepsilon}^{\pm} = \frac{I_\varepsilon}{q_{r\varepsilon}^{\pm}}, \quad \psi_{z\varepsilon}^{\pm} = \frac{I_\varepsilon}{q_{z\varepsilon}^{\pm}}. \tag{4}$$

Substituting (4) into (2) and integrating over the solid angle within the limits of the domains $\omega_r^{\pm}$, $\omega_z^{\pm}$, we obtain

$$\begin{aligned} &\frac{\partial (c_{rr}^{\pm} q_r^{\pm})}{\partial r} + \frac{\partial (c_{rz}^{\pm} q_r^{\pm})}{\partial z} - \frac{S_r^{\pm} q_r^{\pm}}{r} = -k (q_r^{\pm} b_r^{\pm} - 2B), \\ &\frac{\partial (c_{zr}^{\pm} q_z^{\pm})}{\partial r} + \frac{\partial (c_{zz}^{\pm} q_z^{\pm})}{\partial z} - \frac{S_z^{\pm} q_z^{\pm}}{r} = -k (q_z^{\pm} b_z^{\pm} - 2B), \end{aligned} \tag{5}$$

where the subscript $\varepsilon$ has been omitted for convenience in the writing, and the following notation has been introduced:

$$\begin{aligned} &c_{rr}^{\pm} = \int_{\omega_r^{\pm}} \psi_r^{\pm} \sin\theta \cos\varphi d\Omega = 1, \quad c_{rz}^{\pm} = \int_{\omega_r^{\pm}} \psi_r^{\pm} \cos\theta d\Omega, \\ &S_r^{\pm} = \int_{\omega_r^{\pm}} \frac{\partial \psi_r^{\pm}}{\partial \varphi} \sin\theta \sin\varphi d\Omega, \quad b_r^{\pm} = \int_{\omega_r^{\pm}} \psi_r^{\pm} d\Omega, \\ &c_{zr}^{\pm} = \int_{\omega_z^{\pm}} \psi_z^{\pm} \sin\theta \cos\varphi d\Omega, \quad c_{zz}^{\pm} = \int_{\omega_z^{\pm}} \psi_z^{\pm} \cos\theta d\Omega = 1, \\ &S_z^{\pm} = \int_{\omega_z^{\pm}} \frac{\partial \psi_z^{\pm}}{\partial \varphi} \sin\theta \sin\varphi d\Omega, \quad b_z^{\pm} = \int_{\omega_z^{\pm}} \varphi_z^{\pm} d\Omega. \end{aligned} \tag{6}$$

The system (5) is hyperbolic and can be integrated along the characteristic directions

$$\frac{dz}{dr} = c_{zr}^{\pm}, \quad \frac{dz}{dr} = (c_{zr}^{\pm})^{-1}. \tag{7}$$

In a multigroup approximation $k_\varepsilon$ is replaced by a certain mean coefficient $k_g$ in the group (the subscript g) within the limits $\varepsilon_1^g \leqslant \varepsilon \leqslant \varepsilon_2^g$, and the quantity $B_\varepsilon$ is replaced by

$$B_g = \int_{\varepsilon_1^g}^{\varepsilon_2^g} B_\varepsilon d\varepsilon.$$

Shown as an illustration in Fig. 1 are the characteristic directions $c_{zr}^{+}$ and $c_{rz}^{\pm}$ for a group with the limits 10-18.6 eV, as well as the initial distributions of T and $\rho$ found in solving [6] the problem of radiation wave propagation. For characteristic 3-4 eV plasma temperatures, the main part of its thermal radiation is concentrated in this group. The angular distribution of the radiation at each node of the computational mesh was computed for 50 beams for each of six groups. The absorption coefficients were taken from tables [7].

The field $c_{zr}^{\pm}$ and $c_{rz}^{\pm}$ for other groups appears in an analogous way: it does not change its nature even at other times. Meanwhile, a strong change in the characteristic directions related to the change in angular directivity of the radiation is observed in the zone of the highest temperature gradients. As the radiation wave front propagates opposite to the laser radiation (incident from the right) and in the direction of laser pulse propagation, the zone of an abrupt change in the mean cosines shifts together with the front. Simple freezing of the dimensionless coefficients in the averaged equations at the nodes of the computational mesh would result in large errors during passage by the originating wave fronts through this

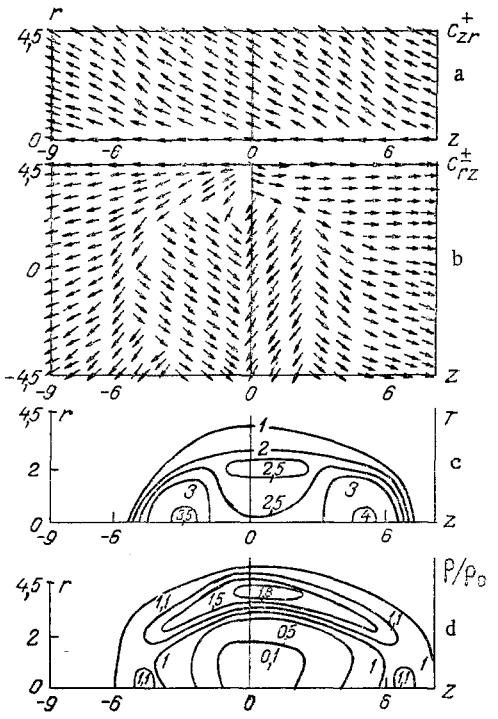


Fig. 1. Characteristic directions $c^+_{zr}$ (a) and $c^\pm_{rz}$ (b) and initial distributions of the temperature T (c) and density ρ (d) in the problem of propagation of radiation originating during absorption of neodymium laser radiation with flux density 100 MW/cm², beam radius 1.5 cm, and initial air density of 1/10 of normal for the time 3 μsec; T, eV.

point if the averaging is not repeated after a short time, which would make the method uneconomical. In [4], it is hence proposed to extract the combinations r, t, the "principal variable" $\omega_g$, related somehow to the motion of the fronts mentioned. In this case the optical thickness calculated for a given group along the characteristic direction and measured from the characteristic points of the heating wave front, e.g., points with a given temperature value, can be used as $\omega_g$. The application of different principal variables for different groups permits extraction of fronts being propagated because of absorption of radiation from different sections of the spectrum (different groups).

The application of a "conversion" permits taking into account, in addition, the change in the dimensionless coefficients in the averaged equations with the time t for fixed values of $\omega_g$, and their comparison at the averaging times $t_k$ and $t_{k+1}$ and in the stages "count" and "convert" affords a possibility of obtaining an objective criterion for the averaging frequency necessary.

2. Averaging with respect to the angles but also with respect to the frequencies was performed in [4] for one-dimensional radiation gasdynamic (RGD) problems. The system (5), which is equivalent to the system of equations of the quasidiffusion method or of the diffusion approximation, can be averaged also in an analogous manner. However, in a number of cases the angular distribution is sufficiently simple and it can be described by using a small set of beams, or it is complex and varies so rapidly that the application of averaging with respect to the angles would be inefficient. In such cases it could be expedient to take the average of the transport equation (1) with respect to the frequencies for each of the selected set of beams [8].

Let us introduce the spectrum of radiation being propagated along a given line and its positive (plus superscript) and negative (minus superscript) directions

$$\varphi^\pm_{\varepsilon g} = I^\pm_\varepsilon / F^\pm_g, \quad F^\pm_g = \int_{\varepsilon_1}^{\varepsilon_2} I^\pm_\varepsilon d\varepsilon. \tag{8}$$

Integrating (1) with respect to ε, we obtain

$$\pm \frac{\partial F^\pm_g}{\partial s} = - \langle k \rangle^\pm_g F^\pm_g + k^P_g \frac{B_g}{\pi}, \tag{9}$$

$$\langle k \rangle^\pm_g = \int_{\varepsilon_1}^{\varepsilon_2} k_\varepsilon \varphi_{\varepsilon g} d\varepsilon, \quad k^P_g = \int_{\varepsilon_1}^{\varepsilon_2} k_\varepsilon \varphi^P_{\varepsilon g} d\varepsilon, \quad \varphi^P_{\varepsilon g} = \frac{B_\varepsilon}{B_g}. \tag{10}$$

We will omit the subscript g on the appropriate quantities in taking the average over the whole spectrum $\varepsilon_1 = 0$ and $\varepsilon_2 = \infty$. In order to characterize the distinction between the true and the Planck spectra (its "distortion"), we introduce the distortion factor

$$\xi_g^{\pm} = \langle k \rangle_g^{\pm} / k_g^P. \tag{11}$$

The method of averaging [4] was applied successfully to the solution of a number of one-dimensional nonstationary problems: a) on the propagation of strong intensively emitting shocks [8-12]; b) on the interaction of laser radiation of different wavelengths with an obstacle in a vacuum, the heating and scattering of vapors being formed with their reradiation taken into account [13]; c) analogous problems for the case of power fast-particle fluxes [14] and thermal radiation impulses [15]; d) on propagation of sub- and supersonic radiation absorption waves in gases opposite to laser radiation [16-18]; e) on plane and spherically symmetric motions of an emitting gas in explosions [19, 20]. The method turned out to be sufficiently effective in all these cases (the averaging is usually made after 200-300 computational time layers). Averaging over the spectrum (with averaging over the angles or with conservation of the individuality of the separate beams) will apparently be efficient even for the solution of two-dimensional problems corresponding to the mentioned one-dimensional problems.

Let us note certain features of the method [4]. The quantity $k_g^P$ characterizes the emissivity of a gas volume element, and the quantity $\langle k \rangle_g$ characterizes the absorption of radiation by this same element. Dependences of the quantity $\xi$ (for averaging over the whole spectrum) in a uniformly heated volume of air on the distance s measured from the boundary of the volume under consideration along a selected direction are presented in [8] for a pressure p. The difference between $\langle k \rangle$ and $k^P$ can be great. Thus, the values of $\xi$ reach $10\text{-}10^2$ for $T = 3 \cdot 10^4$°K and for $s < 1$ cm and $p < 10$ bar. For the case of bulk deexcitation we obtain from (1), (8), and (11)

$$\varphi_{\varepsilon g}^{\pm} = \frac{k_\varepsilon}{k_g^P} \frac{B_\varepsilon}{B_g}, \quad \langle k \rangle_g^{\pm} = \int_{\varepsilon_1}^{\varepsilon_2} k_\varepsilon^2 B_\varepsilon d\varepsilon / (k_g^P B_g). \tag{12}$$

The role of the sections of the spectrum with the highest values of the spectral absorption coefficient $\langle k \rangle_g^{\pm}$ is emphasized still more in the calculation of $k_\varepsilon$ than in the case of the Planck mean. Correspondingly, $\langle k \rangle_g^{\pm} \gg k_g^P$ and $\xi_g^{\pm} \gg 1$. The averaged equation (9) can be written in the form

$$\pm \frac{\partial F_g}{\partial \tau_g^{\pm}} = -\left( F_g^{\pm} - \frac{B_g}{\pi \xi_g^{\pm}} \right), \quad \tau_g^{\pm} = \int_0^s \langle k \rangle_g^{\pm} ds. \tag{13}$$

To obtain a sufficiently exact result in domains with a strong change in $\xi_g^{\pm}$, the introduction of a dense mesh may be required in the numerical integration (13). The peripheral layers bounding a transparent medium or a vacuum always emit radiation in a volume manner. Hence, the quantity $\xi \gg 1$ in these domains can also differ strongly from the value of $\xi_g$ in deeper domains of the space.

In optically thick layers $\xi_g \to 1$, and the passage to the limit to the regime of radiant heat conduction is obtained only by using the series expansion of $\xi_g$. Sometimes the necessity to modernize the method of [4] occurs for the case of an optically thick plasma.

We denote the magnitudes of the radiation intensity for the positive and negative directions of the same line by the superscripts + and −. The transport equations become

$$\frac{\partial H_\varepsilon}{\partial s} = -k_\varepsilon (Z_\varepsilon - 2B_\varepsilon), \quad \frac{\partial Z_\varepsilon}{\partial s} = -k_\varepsilon H_\varepsilon, \quad H_\varepsilon = I_\varepsilon^+ - I_\varepsilon^-, \quad Z_\varepsilon = I_\varepsilon^+ + I_\varepsilon^-. \tag{14}$$

Integrating them with respect to the frequency, we obtain

$$\frac{\partial H_g}{\partial s} = -(k_g^Z Z_g - 2k_g^P B_g), \quad \frac{\partial Z_g}{\partial s} = -k_g^H H_g,$$
$$k_g^Z = \int_{\varepsilon_1}^{\varepsilon_2} k_\varepsilon Z_\varepsilon d\varepsilon / Z_g, \quad k_g^H = \int_{\varepsilon_1}^{\varepsilon_2} k_\varepsilon H_\varepsilon d\varepsilon / H_g, \tag{15}$$

$$Z_g = \int_{\varepsilon_1}^{\varepsilon_2} Z_\varepsilon d\varepsilon,\ H_g = \int_{\varepsilon_1}^{\varepsilon_2} H_\varepsilon d\varepsilon.$$

In the regime of radiant heat conductivity

$$Z_\varepsilon \approx 2B_\varepsilon,\ Z_g \approx 2B_g,\ H_\varepsilon \approx -\frac{1}{k_\varepsilon}\frac{\partial B_\varepsilon}{\partial s},\ H_g \approx -\frac{1}{k_g^R}\frac{\partial B_g}{\partial s},$$
$$k_g^Z \approx k_g^P,\ k_g^H \approx k_g^R,\ l_g^R = \frac{1}{k_g^R} = \left[\int_{\varepsilon_1}^{\varepsilon_2} \frac{\partial B_\varepsilon}{\partial T}\frac{1}{k_\varepsilon}\,d\varepsilon\right]/(dB_g/dT). \tag{16}$$

Let us note that those sections of the spectrum where $k_\varepsilon$ are small introduce the main contribution to the quantity $k_g^R$. The equations (15) can be called the equations of quasi-diffusion along a beam. Their boundary conditions are given in the form of combinations of $Z_\varepsilon$ and $H_\varepsilon$, which requires the application of factorization; however, there is no necessity when solving elliptic equations which occur during averaging over the angle. In general, variability in the sign of $H_\varepsilon$ can hold, which will result in large differences in $k_g^H$ for a small change in $H_\varepsilon$, and such an averaging method will apparently be sufficiently effective only in regimes close to radiant heat conduction. The "flux" equations (9) for quantities with the superscripts + and -, whose integration is performed independently in opposite directions, disclose the specifics of radiation transport best under strong nonequilibrium conditions. Since the optical thicknesses and the nature of the temperature distribution can be different along different beams passing through a given point, (9) can be used in some directions, and (15) in others.

3. On the section $\Delta S = s_2 - s_1$ with optical thickness $\Delta\tau_\varepsilon = \tau_\varepsilon^2 - \tau_\varepsilon^1$ let the quantity $B_\varepsilon$ vary between points 1 and 2 in a linear manner:

$$B_\varepsilon = B_\varepsilon^1 + \frac{dB_\varepsilon}{d\tau_\varepsilon}(\tau_\varepsilon - \tau_\varepsilon^1),\ \frac{dB_\varepsilon}{d\tau_\varepsilon} = \frac{B_\varepsilon^2 - B_\varepsilon^1}{\Delta\tau_\varepsilon}. \tag{17}$$

Then the exact solution of (1) in the interval $s_1 \leqslant s \leqslant s_2$ has the form

$$I_\varepsilon^2 = B_\varepsilon^2 + (I_\varepsilon^1 - B_\varepsilon^1)E_\varepsilon - \frac{dB_\varepsilon}{d\tau_\varepsilon}(1 - E_\varepsilon),\ E_\varepsilon = \exp(-\Delta\tau_\varepsilon). \tag{18}$$

The superscripts 1 and 2 denote values of the quantities on the interval boundary. Here (18) is used in calculating $I_\varepsilon$, then integration is with respect to $\varepsilon$, then $F_g$, the radiation spectrum $\Phi_\varepsilon$, and the coefficient $\langle k\rangle_g$ in (9) are found, and the average equation is again solved for $F_g$ by using some difference scheme. It is meanwhile possible to take the average of the difference equation directly. Such an idea has already been proposed in different forms [21-24]. The averaging in [21] was with respect to the frequency for diffusion-type difference equations. The coefficients in the averaged equations are assumed frozen. It is noted that freezing the solution (freezing the spectrum in our terminology) yields the best results. Averaging with respect to the angle and frequency of the flux-type difference equations is performed in [22-24]. We perform an analogous averaging of the difference equation (18) - the analog of (1) written along an individual direction. We consequently obtain

$$F_g^2 = F_g^1 E_g^1 + B_g^2 - B_g^1 A_g^1 - \frac{(B_g^2 - B_g^1)L_g}{\Delta s},$$
$$E_g^1 = \int_{\varepsilon_1}^{\varepsilon_2} I_\varepsilon^1 E_\varepsilon d\varepsilon / F_g^1,\quad A_g^1 = \int_{\varepsilon_1}^{\varepsilon_2} B_\varepsilon^1 E_\varepsilon d\varepsilon / B_g^1, \tag{19}$$
$$\Delta\tau_\varepsilon = (k_\varepsilon^2 + k_\varepsilon^1)\Delta s/2.$$

For large optical thicknesses ($\Delta\tau_\varepsilon \gg 1$) and a small change in temperature ($B_\varepsilon^2 \approx B_\varepsilon^1$) the characteristic distance $L_g$ goes over into $l_g^R$:

$$L_g = \frac{1}{2}\int_{\varepsilon_1}^{\varepsilon_2} \frac{dB_\varepsilon}{dB_g}\left(\frac{1}{k_\varepsilon^2} + \frac{1}{k_\varepsilon^1}\right)d\varepsilon = \frac{1}{2}\left(\frac{1}{(k_g^R)_2} + \frac{1}{(k_g^R)_1}\right) = \frac{1}{2}[(l_g^R)_2 + (l_g^R)_1]. \tag{20}$$

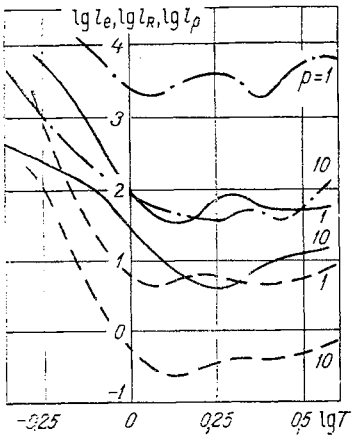


Fig. 2

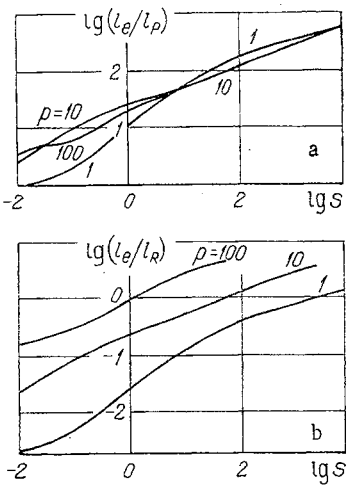


Fig. 3

Fig. 2. Dependence of the magnitudes of the paths $l$, cm, on the temperature T, eV, at pressures p = 1 and 10 bar. The effective paths $l_e$ for thickness s = 1 cm are given by the continuous curves, the Planck paths $l_P$ are given by the dashed curves, and the Rosseland paths $l_R$ are given by the dash-dot curves.

Fig. 3. Dependences of the ratio between the paths $l_e$ and $l_P$ (*a*), $l_e$ and $l_R$ (b) in air for $T = 3 \cdot 10^4$°K for different pressures p (the values of p, bar, are indicated at the appropriate curves) on the distance s, cm.

For small optical thicknesses, $L_g^2 \approx \Delta s$. The effective optical thicknesses and their corresponding effective absorption factors can be introduced by starting from the following:

$$\exp(-\tau_g^A) = A_g^1, \quad k_g^A \Delta s = \tau_g^A, \quad \exp(-\tau_g^E) = E_g^1, \quad k_g^E \Delta s = \tau_g^E,$$
$$1 - \exp(-\tau_g^s) = \frac{L_g}{\Delta s}\tau_g^s, \quad k_g^s \Delta s = \tau_g^s. \tag{21}$$

Because of (19), we can write in a form similar to (18):

$$F_g^2 = F_g^1 \exp(-\tau_g^E) + B_g^2 + B_g^1 \exp(-\tau_g^A) + [(B_g^2 - B_g^1)/\tau_g^s][1 - \exp(-\tau_g^s)]. \tag{22}$$

For large optical thicknesses $\tau_g^s = \Delta s/L_g$, $k_g^s = 1/L_g = k_g^R$, while for small we obtain $\tau_g^s = 2L_g/\Delta s$, $k_g^s = 2/L_g = 2/\Delta s$. Let us introduce

$$\Lambda_g^A = k_g^A/k_g^P, \quad \Lambda_g^E = k_g^E/k_g^P, \quad \Lambda_g^s = k_g^s/k_g^R. \tag{23}$$

It was proposed to freeze coefficients of the type $A_g^1$ and $E_g^1$ in [22] in intervals between the averages. However, this can turn out to be inconvenient, e.g., in the case when the size of the cell, and the optical thickness as well, vary strongly, but the mean absorption factor itself varies slightly. In conformity with the general idea of using averaged equations [4], it is proposed in [24] to use dimensionless coefficients of the distortion coefficients type ($\Lambda_g^A$, $\Lambda_g^E$ and $\Lambda g$), where they are not frozen, but are interpolated with respect to time and the principal variable. It is expedient to do this even in the two-dimensional case. In problems where moving zones exist for sharp change in the temperature, radiation spectra, and fluxes — "fronts" — and they are quite definite, it is possible to take $s - s_g^f$ as $\omega_g$, where $s_g^f$ is the coordinate of the front on the given beam.

Let us consider radiation emission and absorption by a uniformly heated layer of gas. Integrating (1) with respect to the frequency, finding the quantity $F_g$, we determine the effective paths $l_e$ in such a way that the expression written in a form analogous to a gray gas would yield the correct value of the quantity $F_g = B_g(1-\exp(-s/l_e) + F_g^0\exp(-s/\bar{l}_e)$. The quantities $l_e$ and $\bar{l}_e$ will naturally depend on the length s. As is seen from Fig. 2, dependences of the paths $l_e$, $l_P$ and $l_R$ on the temperature T are qualitatively identical,

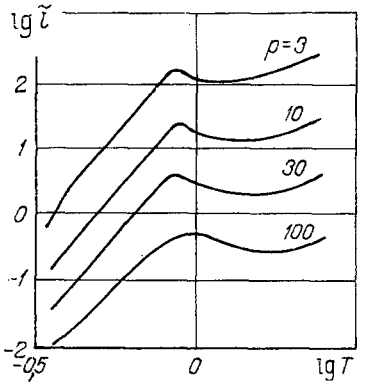


Fig. 4. Dependence of the quantity $\bar{l}$, cm, the mean path characterizing the degree of penetration of the external radiation source (with a temperature of $T = 3 \cdot 10^4$°K), on the temperature T, eV, of an absorbing air layer.

but quantitatively the difference is great. A comparison of the quantities $l_e$ and $l_p$, $l_e$ and $l_R$ displayed in Fig. 3 shows that the difference can reach $10^2$-$10^3$ times. Therefore, the result of an exact solution of the spectral problem can differ radically from the solution for a gray gas. A multigroup approximation usually improves the situation; however, as experience in solving the one-dimensional problems shows [8-21], even when applying 10-15 groups the results of the solution can differ two- to threefold from the solution of the spectral problem. Meanwhile, utilization of such a number of groups (in a multigroup approximation or by using exact averaging) makes obtaining the solution quite too tedious. Hence, it is desirable to average over a minimal number of groups with the greatest step in space.

The effective coefficient $k_e = l_e^{-1}$, which is analogous in meaning to the coefficient $k^A$, agrees with it for a uniformly heated gas layer. The difference is just that when using $l_e$ to estimate the emissivity, the size of the whole emitting volume appears as the characteristic dimension, while the size of the elementary cell $\Delta s$ emerges as such a characteristic dimension in numerical computations using averaging. As Fig. 2 shows, the dependence $l_e(s)$ is comparatively weak. It is possible to select $k_e$ rather than $k^P$ for the reference absorption factor in expressions of the type (23) for $\Lambda_g^A$. Let us note that the Rosseland mean path also entered the averaged equations in a natural manner. When using averaging of the difference equation in one-dimensional problems of the type [13], averaging was successfully performed with respect to just one group. Averaging of the transport difference equations yields better results than the method of averaging the original differential equation when the temperature profile is close to the approximate value used in the construction of the difference scheme, and the transport equation can be integrated by a large step in space. Let us note that a parameter distribution close to the isothermal is encountered sufficiently frequently in problems with strong radiant heat exchange, which results in temperature equilibration.

The radiation spectrum of external sources can differ radically from the "intrinsic" radiation spectrum within the volume under consideration. The dependences of $\bar{l}$ on T are qualitatively different in nature as compared with the dependences $l_e(T)$ (Fig. 4). Hence, it is expedient to take the average separately for the external and the intrinsic radiation. Let us note that the quantity $k = \tilde{l}^{-1}$ is analogous to $k^E$. The advantages of averaging the difference rather than the differential equations are less obvious in the case when the characteristic process is not emission but absorption of the radiation of an external source.

Stationary problems of the flow around bodies with a substantial radiation role are also of great interest. The method of buildup or the iteration method are ordinarily used for the solution. In this case the method of averaging can evidently also be used. The iteration or buildup processes can here be performed for the "frozen" coefficients in the averaged equations, and then averaging should again be performed and duplication of the process occurs. If the solution is executed for many nearby variations, then in a first approximation the coefficients in the averaged equations can be taken on the basis of the results of computing the preceding variations.

The ideas of the method in [4] are extendable to the case of a nonequilibrium state of a substance.

## NOTATION

h, Planck constant; $\nu$, frequency; $\varepsilon = h\nu$, photon energy; $I_\varepsilon$, radiation intensity with photon energy $\varepsilon$ referred to unit interval of quantum energy; s, distance along a beam whose

direction is characterized by the unit vector $\Omega$; $k_\varepsilon$, linear spectral absorption coefficient with a correction for stimulated emission; $B_\varepsilon$, Planck function, $\sigma$, Stefan–Boltzmann constant; T, temperature; r, z, cylindrical coordinates; $\theta$, angle between the beam and the z axis; $\varphi$, angle between the projections of the radius-vector of the point (r, z) and the projection of the beam on the plane z = 0; $d\Omega$, solid-angle element; t, time, $F_g^{\pm}$, integrated (with respect to the frequency) radiation intensity; $\langle k\rangle_g^{\pm}$, absorption factor averaged over the true spectrum at a given point; $k_g^P$, Planck mean absorption factor for a given group; $\ell_g^R$, mean Rosseland radiation path; $k_g^R$, mean absorption factor corresponding to $\ell_g^R$.

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# AN EXPERIMENTAL INVESTIGATION OF EGD FLOW OF GAS IN A CORONA DISCHARGE AND ITS INFLUENCE ON THE MOTION OF DISPERSED PARTICLES

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The procedure and results of an experimental investigation of the EGD gas flow accompanying a corona discharge — the electric wind — are presented. It is shown that the motion of dispersed particles in a corona discharge is determined mainly by the electric wind.

The motion of gas-dispersed streams in the field of a corona discharge takes place in a number of technological devices. Such devices include electric filters, electrostatic separators, devices for depositing various coatings, etc. The processes taking place in such devices are extremely complicated, and their description is often associated with great mathematical difficulties. Therefore, experiment still remains the principal method of investigating these processes.

The most often encountered method of analyzing the behavior of dispersed particles in the field of a corona discharge, placed at the foundation of the design of such apparatus, is based on the determination of the Coulomb forces acting on particles charged through ion sorption and the forces of hydraulic drag (Stokes forces) [1-3]. In the simplest form this approach yields the following equation of motion of a spherical charged particle in an electric field:

$$m\frac{d\bar{v}}{dt} = q\bar{E} - 6\pi\mu a\bar{v} \pm m\bar{g}. \qquad (1)$$

Different variants of Eq. (1) are possible, allowing for nonuniformity of the electric field, variability of the particle charge, etc. However, such an approach does not allow for the influence on the motion of the dispersed particles of the EGD flow accompanying a corona discharge in gases — the electric wind. The mechanism of its generation is this. In a unipolar corona discharge, when the corona-forming electrode is the cathode, negative gas ions are formed near the latter. Under the action of the electric field they move toward the oppositely charged electrode and, in the process of motion, in colliding with the neutral gas molecules they impart kinetic energy to them: $\varepsilon_i = m_i\bar{v}_i^2/2$.

The energy obtained by a neutral molecule in a collision with an ion is determined by the relation $\varepsilon_n = (m_n m_i^2(1-\cos\theta)\bar{v}_i^2/(m_i + m_n)^2$.

In air, in particular, the masses of the $O_2^-$, $O_3^-$ and other ions formed in a corona discharge are close to the mass of a neutral molecule. Therefore, an estimate of the efficiency of kinetic-energy transfer provides a basis for assuming that in each collision act, the energy of an ion is fully transferred to a neutral molecule. The collision frequency is determined by the gas density and the ion velocity, which depends in turn on the gas temperature and the magnitude of the electric field. Mass motion of the gas directed away from the

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